Extracting Energy from Accretion into Kerr Black Hole

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ABSTRACT

The highest efficiency of converting rest mass into energy by accreting matter into a Kerr black hole is $\sim 31\%$ (Thorne 1974). We propose a new process in which periods of accretion from a thin disk, and the associated spin-up of the black hole, alternate with the periods of no accretion and magnetic transfer of energy from the black hole to the disk. These cycles can repeat indefinitely, at least in principle, with the black hole mass increasing by $\sim 66\%$ per cycle, and up to $\sim 43\%$ of accreted rest mass radiated away by the disk.

Subject headings: black hole physics — accretion disks — magnetic fields

1. Introduction

Accretion of matter from a thin disk into a black hole may spin up the black hole up to $a^\star = 0.998$, at which point $\sim 31\%$ of the rest mass of accreted matter is radiated away (Thorne 1974). We define $a^\star \equiv ca/GM_H$, where M_H is the black hole mass and a is its angular momentum per unit mass. Energy may be extracted out of a rapidly spinning black hole by means of the Blandford-Znajek mechanism and transported to a distant load by means of magnetic field (Blandford & Znajek 1977). Recently it has been shown that even more efficient extraction of rotational energy is possible with the magnetic field lines threading the Kerr black hole and connecting it to the inner parts of the disk (Li 2000). In these two processes energy of black hole rotation may be extracted with up to 9% and 15% efficiency, respectively (Li 2000).

In this letter we propose an efficient way of converting rest mass to energy by alternating the two processes: accretion of matter from a thin disk into a Kerr black hole and the magnetic extraction of energy from the black hole to the disk. We show that this process has the highest efficiency in converting mass into energy through accretion into a black hole among those have been ever suggested: up to $\sim 43\%$ of accreted rest mass is radiated away by the disk, with the black hole mass increasing by $\sim 66\%$ per cycle.

2. Results

We consider a black hole with the initial mass M_{A1} and spin $a^{\star}_{A1}$. Next, the black hole is spun up by matter accreting from a thin, Keplerian disk; its mass and spin increase to M_{B1} and $a^{\star}_{B1}$, respectively. During this phase the amount of accreted disk rest mass is M_{d1}, and the total energy E_{A1} is radiated by the disk. Next the accretion stops, and the black hole transfers angular momentum and energy to the disk by means of magnetic field, as described by Li (2000), reducing the mass and the spin to the new values M_{A2} and $a^{\star}_{A2}$, and transferring energy E_{B1} to the disk. That energy is radiated by the disk, while no new matter is accreted into the black hole.

During the evolution from A_1 to B_1 we calculate the change of black hole mass and spin, as well as the efficiency of converting accreted mass to radiation, using equations [3] and [4] provided by Bardeen (1970). In this calculation we have used Bardeen's simple solutions for accretion and the effects of 'photon capture' of Thorne (1974) have not been taken into account, except that $a^{\star}$ has been limited to values not exceeding $a^{\star}_{max} = 0.998$. Note that the efficiency of converting rest mass to radiation by accretion from a thin disk into $a^{\star} = 0.998$ Kerr black hole is equal to 32.4% in Bardeen's formulation, but it is only 30.8% in the Thorne's model. However, this difference becomes insignificant when the $a^{\star}$ parameter is somewhat smaller than 0.998. Therefore, we overestimate the efficiency of energy release in during the evolution from A_1 to B_1 by a small amount only. During the evolution from B_1 to A_2 we use equation [13] as given by Li (2000).

We consider a cycle which could be repeated indefinitely, and therefore we seek $a^{\star}_{A1} = a^{\star}_{A2}$. We would like the process to be as efficient as possible in converting the accreted disk mass M_{d1} to the radiated energy $E_1 = E_{A1} + A_{B1}$. Hence, we seek the values of $a^{\star}_{A1}$ and $a^{\star}_{B1}$ which maximize the ratio $E_1/M_{d1}c^2$. It turns out that the optimum values are: $a^{\star}_{A1} = a^{\star}_{A2} \approx 0.3594$, and $a^{\star}_{B1} \approx 0.998$, which give the efficiency of converting disk rest mass to radiation: $\eta_{max} = E_1/M_{d1}c^2 \approx 0.436$.

The evolution of our system is shown in Fig. 1: the black hole mass M_H varies as a function of integrated amount of energy radiated from the disk E. Obviously, E varies monotonically with time, but M_H increases during the accretion phase (from A_1 to B_1), and it is reduced during the spin-down phase (from B_1 to A_2). The black hole mass increases by the same factor in every cycle, and it can grow as long as the cycles continue.

During the first part of the cycle the black hole mass increases by a factor 1.9625 between points A_1 and B_1, while the amount of rest mass accreted from a thin disk is equal to $1.1788 M_{A1}$, where M_{A1} is the initial black hole mass. The balance, equal to $0.2163 M_{A1}c^2$ is radiated away. In the second part of the cycle no mass is accreted, the black hole mass is reduced to 1.6644 of its initial value, and the corresponding amount of energy, i.e. $0.2981 M_{A1} c^2$ is transferred to the disk and radiated away. During the complete cycle the black hole mass increases by a factor 1.6644, and the total energy radiated from the disk is equal to $E_1 = 0.5144 M_{A1} c^2 = 0.436 M_{d1} c^2$, where M_{d1} is the total accreted disk mass.

This process can be repeated indefinitely, at least in principle. In each cycle the black hole mass increases by a factor 1.6644. The system evolves through alternating phases of disk accretion associated with the black hole spin-up, and phases of no accretion and the disk radiating away energy transferred from the spinning-down black hole by means of the magnetic field. The evolution of the system through two full cycles is illustrated in Fig. 1.

3. Conclusions

It is far from clear if the scenario we propose can be maintained over many cycles, or even over one cycle. However, it shows that it may be possible, at least in principle, to radiate up to $\sim 43\%$ of rest mass energy, with only $\sim 57\%$ ending up permanently in the black hole. This is the highest efficiency of converting rest mass to energy by accreting into a black hole that has been ever proposed.

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Fig. 1.— The evolution of the black hole - disk system is shown through two full cycles of the accretion - spin-down process, with time increasing from left to right. The black hole mass M_H in units of the initial mass M_{A1} is shown as a function of the integrated amount of energy radiated by the disk, in units of $M_{A1}c^2$. Each cycle contains two phases: phase $A \rightarrow B$, during which the black hole mass and spin increase due to accretion from a thin disk, and phase $B \rightarrow A$, during which there is no accretion and the black hole transfers energy and angular momentum to the disk. In every cycle the black hole mass increases by a factor ~ 1.66, and $\sim 43\%$ of the rest mass of accreted matter is radiated away.